

Conditional Translation Problems D1, D2: Discussion

Assignment: Translate each of the following English sentences into the formal language of Chapter Three.

D1. If either Rex or Ace cheats on the final exam, then the fraternity will be put on probation and the formal dance will be cancelled.

D2. Either Rex will study for the exam, or he'll pass only if the exam is very easy.

Discussion:

D1. The first step in translating into formal language is to pick out all of the logical form phrases in the sentence.

If either Rex or Ace cheats on the final exam, then the fraternity will be put on probation and the formal dance will be cancelled.

Here the form phrases are “if... then,” “either... or” and “and”.

If either Rex **or** Ace cheats on the final exam, **then** the fraternity will be put on probation **and** the formal dance will be cancelled.

What's left after picking out the form phrases are our candidates for subject matter sentences (which will each receive a sentence letter). We make a list of these subject matter sentences.

Note that while “Ace cheats on the exam” is a complete sentence, “Rex” is not a sentence.

Rex
Ace cheats on the exam

But clearly what's happened here is that there are two sentences which share some (repeated) material, and English has chopped the repeated material out of the first sentence.

The material that should show up in both sentences is “cheats on the exam”. Restoring this repeated material to the first sentence, we get two complete subject matter sentences, and give each its own sentence letter.

P: Rex [cheats on the exam]

Q: Ace cheats on the exam

The other two subject matter sentences here are complete as they stand.

If either Rex **or** Ace cheats on the final exam, **then** the fraternity will be put on probation **and** the formal dance will be cancelled.

P: Rex cheats on the exam.

Q: Ace cheats on the exam.

R: The fraternity will be put on probation.

S: The formal dance will be cancelled.

With the translation table complete, we replace the subject matter sentences of English with their matching sentence letters.

If either Rex **or** Ace cheats on the final exam, **then** the fraternity will be put on probation **and** the formal dance will be cancelled.

If either P or Q, then R and S.

Now we only need to translate the form phrases into formal language. We have three form phrases competing here to be the main connective of the sentence. A clue for figuring out which phrase wins is the comma: the comma marks a main break in the sentence, which is a likely place for the main connective to sit. Here the word “then” is sitting right in that comma gap – suggesting that “if... then” is the main connective.

If either P or Q, | then R and S.

“If... then” is of course translated by the arrow. “If” is an ordinary conditional phrase (no “only”), and so comes right before the consequent.¹ That means “either P or Q” is the antecedent; so “R and S” must be the consequent.

If either P or Q, then R and S.

(either P or Q \rightarrow R and S)

“Either... or” is translated by the vel.

(either P or Q \rightarrow R and S)

((P \vee Q) \rightarrow R and S)

“And” is translated by the wedge.

((P \vee Q) \rightarrow R and S)

((P \vee Q) \rightarrow (R \wedge S))

This completes the translation.

1. If either Rex or Ace cheats on the final exam, then the fraternity will be put on probation and the formal dance will be cancelled.

P: Rex cheats on the exam.

Q: Ace cheats on the exam.

R: The fraternity will be put on probation.

S: The formal dance will be cancelled.

((P \vee Q) \rightarrow (R \wedge S))

¹ “Then” is just an optional companion – as we know from cases of “if” without a “then”.

D2. Once again we begin by picking out the form phrases of the sentences.

Either Rex will study for the exam, or he'll pass only if the exam is very easy.

Here there are two form phrases: “either... or” and “only if”.

Either Rex will study for the exam, **or** he'll pass **only if** the exam is very easy.

What's left beyond these form phrases are the three subject matter sentences “Rex will study for the exam,” “Rex will pass” and “The exam is very easy”. A translation table gives each of these a sentence letter.

P: Rex will study for the exam

Q: Rex will pass.

R: The exam is very easy

Replacing the subject matter sentences with matching sentence letters yields this.

Either P, or Q only if R

Since “or” appears by the comma, we assume “either... or” is the main connective. It's translated by the vel (with matching outer parentheses).

(P \vee Q **only if** R)

The right part of this disjunction – “Q only if R” – is a conditional. “Only if” is translated by the arrow.

Q only if R

(\rightarrow)

Since “only if” is an ‘**only**’ phrase, our Conditional Translation Rule says it comes **right before the consequent**. So “R” must be the consequent, going after the arrow.

$$\begin{array}{c} \underline{Q} \text{ only if } \underline{R} \\ \quad \quad \quad \curvearrowright \\ (\rightarrow R) \end{array}$$

That means “Q” must be the antecedent, going before the arrow.

$$\begin{array}{c} \underline{Q} \text{ only if } \underline{R} \\ \\ (Q \rightarrow R) \end{array}$$

This conditional forms the right part of the larger disjunction.

$$\begin{array}{c} (P \vee (Q \text{ only if } R)) \\ \\ (P \vee (Q \rightarrow R)) \end{array}$$

That completes the translation.